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Impedance Transformations for the Generalized Reflection Modulator

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***Abstract*— A procedure is given for obtaining the impedance transformer which will produce a prescribed pair of phasor reflection coefficients by transformation from a given pair of impedance states terminating the transformer. The transformer may be used in a general two-state reflection modulator: as a phase shifter with phase change at constant amplitude or as an amplitude modulator with level shift at constant or variable phase. The condition governing the reflection coefficients available from a given pair of impedance states is given. Examples and limitations are discussed.**

I. INTRODUCTION

THE REFLECTION modulator is useful as a phase shifter and as an amplitude modulator for digital modulation [1]–[8]. In typical applications, the modulator may be composed of a circulator or hybrid coupler terminated in a pair of variable semiconductor impedance elements. The transmission factor of the modulator is then proportional to the reflection coefficient of the terminations. The application of impedance transformation to modify the reflection coefficients of the terminating impedances was introduced by Kawakami [9] and by Kurokawa and Schlosser [10]. In the present paper, a unified treatment of the reflection phase shifter and reflection-type amplitude modulator is provided. A procedure is given for the calculation of the design parameters of a lossless impedance transformer to produce a specified pair of reflection coefficient values from a given pair of impedance states of the terminating impedances. The condition is given which determines the reflection coefficients that are available from any pair of impedance states. This condition is shown to coincide with the definition of the Q^2 of the terminating impedance pair introduced by Kurokawa and Schlosser.

The switching performance of the reflection modulator depends inherently upon the impedance levels presented at its reflecting terminations. The impedance characteristics of semiconductor devices available for use are significant for this application. The silicon p-i-n diode is frequently employed in switching circuits because its impedance states approach nearly ideal metal switch contact behavior. Interest is presently growing in the adoption of GaAs semiconductor devices in microwave circuits. It is, therefore, relevant to determine whether impedance properties typical of GaAs devices are useful in the reflection modulator circuits being considered. Typical measured values of GaAs Schottky beam-leaded diode impedances at 10 GHz are shown in Table I.

TABLE I
MEASURED IMPEDANCES OF GaAs SCHOTTKY DIODES AT 10 GHz

Diode Current (mA)	Impedance (Ohms)
0.0	6.-j51.
0.5	17.-j45.
1.0	29.-j34.
2.0	35.-j11.

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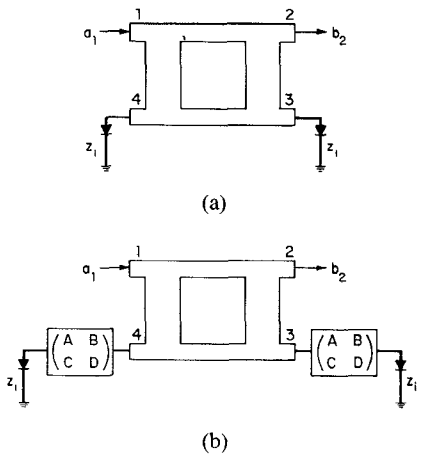

Fig. 1. Hybrid-coupled impedances. (a) Direct terminations. (b) Impedance-transformed terminations.

Because of its high impedance in all bias states, the Schottky diode is not applicable as a simple series or shunt switch analogous to the silicon p-i-n diode. Useful reflection modulators can be constructed using nonideal semiconductors, however, by introducing impedance transformation ahead of the semiconductor termination (Fig. 1). It will be assumed in the following that the terminating impedances of the reflection modulator are operated between a selected pair of impedance states designated Z_1 and Z_2 which can be reached at will by the application of suitable biases. In the case of the digital switching (amplitude) modulator, ideal absorption–reflection (A/R) switch characteristics are wanted. The ideal A/R switch is a two-port junction having scattering matrices in the OFF and ON states, respectively, of the form

OFF: ON:

$$S=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad S=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

The semiconductor devices terminating an A/R switch must be capable of dissipating the entire signal power being transferred.

The diode impedance states shown in Table I for bias currents of 2.0 and 0.0 mA have reflection coefficients of 0.22 and 0.89, respectively. If used as a reflection modulator, their switching matrices would have the magnitudes

OFF: ON:

$$S=\begin{pmatrix} 0. & 0.22 \\ 0.22 & 0 \end{pmatrix} \qquad S=\begin{pmatrix} 0. & 0.89 \\ 0.89 & 0. \end{pmatrix}.$$

It was suggested in 1965 by Kawakami [9] that the off-state isolation of a switching modulator could be improved by impedance matching the off-state impedance to the system characteristic impedance Z_0. If the OFF impedance state Z_1 is transformed to Z_0, ($g_1=0$), it can be shown that the magnitude of the reflection coefficient of impedance state Z_2 transformed by the same impedance transformer is

$$|g_2(Z_2')|=\left|\frac{Z_2-Z_1}{Z_2+Z_1^*}\right| \tag{1}$$

where Z_1^* is the complex conjugate of Z_1. This expression (1) remains invariant under transformation of the impedances by any lossless, reciprocal network. Kawakami, therefore, proposed it as a figure of merit for the impedance state pair Z_1, Z_2. Equation (1) predicts a transformed reflection coefficient magnitude $|g_2|$ of 0.86 in the example above. For some applications, an arbitrarily selected reflection coefficient pair may be required. In the following sections it is shown what pairs of reflection coefficients (g_1, g_2) may be obtained from a given pair of impedance states (Z_1, Z_2) through transformation by a lossless transformer network and how the design parameters of the network may be determined.

II. Impedance Transformer Network Design

The derivation of the design parameters for the impedance-transforming network is based on the expression for the transformed reflection coefficient $g_i(Z_i')$ appearing at the input to the network when it is terminated by impedance state Z_i ($i=1,2$)

$$g_i(Z_i')=\frac{Z_i'-Z_0}{Z_i'+Z_0} \qquad (i=1,2) \tag{2}$$

where Z_0 is the system characteristic impedance, assumed to be real. If the impedance transformer is described by a generalized network (A, B, C, D) matrix, the transformed impedance is given by

$$Z_i'=\frac{AZ_i+B}{CZ_i+D} \qquad (i=1,2). \tag{3}$$

When (3) is used in (2), the result is

$$g_i=\frac{(A-CZ_0)Z_i-(DZ_0-B)}{(A+CZ_0)Z_i+(DZ_0+B)} \qquad (i=1,2). \tag{4}$$

When the transformation network is lossless, the elements (A, B, C, D) of the network matrix reduce to (a, jb, jc, d) where a, b, c, and d are pure-real quantities. For the lossless network, (4) becomes

$$g_i=\frac{(a-jcZ_0)Z_i-(dZ_0-jb)}{(a+jcZ_0)Z_i+(dZ_0+jb)} \qquad (i=1,2). \tag{5}$$

Defining new quantities Z_m and η, (5) may be simplified to

$$g_i=\frac{Z_i-Z_m}{Z_i+Z_m^*}\eta \qquad (i=1,2) \tag{6}$$

where $Z_m=(dZ_0-jb)/(a-jcZ_0)$, Z_m^* is the complex conjugate of Z_m, and η is a phase factor of unit amplitude and angle $\theta=-2\tan^{-1}(cZ_0/a)$. In (6), Z_m is an impedance parameter equal to the impedance that would be transformed to Z_0 by the network, for which case the reflection coefficient g vanishes.

When (6) is written for terminations Z_1 and Z_2, the two cases may be combined by eliminating the common factor η, resulting in a solution for $Z_m=R_m+jX_m$ of the form

$$Z_m=-\alpha\pm\sqrt{\alpha^2+\beta} \tag{7}$$

where α and β are functions of Z_1, Z_2, g_1, and g_2 (see Appendix). The two-parameter description of the network in terms of R_m and X_m is employed here instead of a description in terms of the four A, B, C, D parameters of the network. The expressions are thereby simplified and it is a somewhat more convenient task to design a network to match a given impedance Z_m than to reproduce a given $(ABCD)$ matrix. This restricts the choice of matching circuits to those which can be described in terms of two parameters. There is a sufficient range of two-parameter circuits to satisfy most matching requirements [8].

III. Uniqueness Condition for Matching Network

Unique and well-defined solutions for Z_m in (7) are not available for all arbitrarily chosen values of g_1 and g_2 with given impedance states Z_1 and Z_2. The allowable values of g_1 and g_2 are restricted by a condition which arises from (5). When (5) is written for cases $i=1$ and $i=2$ and the real and imaginary parts separated, the result is a set of four homogeneous equations in the unknown network parameters a, b, c, and d. These equations will have a solution only if the determinant of their coefficients vanishes.[1] The vanishing of the determinant leads to the relation (see Appendix)

$$\frac{|\sigma_1-\sigma_2|^2}{\sigma_{1r}\sigma_{2r}}=\frac{|Z_1-Z_2|^2}{R_1R_2} \tag{8}$$

where σ_1 and σ_2 are reflection variables defined by

$$\sigma_1=\frac{g_1-1}{g_1+1} \qquad \sigma_2=\frac{g_2-1}{g_2+1}. \tag{9}$$

In (8), σ_1, σ_2, g_1, and g_2 are phasor quantities in general and σ_{1r} and σ_{2r} are the real parts of σ_1 and σ_2, respectively.

When (8) is rewritten in terms of g_1 and g_2 it takes the form

$$\frac{4|g_1-g_2|^2}{(|g_1|^2-1)(|g_2|^2-1)}=\frac{|Z_1-Z_2|^2}{R_1R_2}=Q^2. \tag{10}$$

This expression (10) is identical with a relation found by Kurokawa and Schlosser [10], who defined it as the square of the Q of the two-state impedance pair Z_1, Z_2. These authors proposed (10) as the specification of a quality factor for a switching diode which presents impedance states Z_1 and Z_2. In the present analysis, (10) is seen also to be a necessary condition for the existence of a lossless network to generate the transformed reflection coefficients g_1 and g_2 from Z_1 and Z_2. Kurokawa and Schlosser cited the importance of the assumed lossless character of the transformer network in determining (10). The assumption of losslessness is seen here to limit to four the number of parameters (a, b, c, d) needed to describe the circuit. Since only four simultaneous equations are available from the separation of the real and imaginary parts of cases $i=1$ and $i=2$ of (5), this enables (5) to yield the system determinant of the equations, providing the existence condition (8) and (10). The assumption of a lossless transformation network is not a serious restriction since in microstrip or waveguide format the network will normally be constructed of sections of microstrip or waveguide transmission line for which the assumed lossless propagation is an acceptable approximation.

The necessary condition (10) implies a specific relationship between Z_1, Z_2, g_1, and g_2. Therefore, it is important to avoid specifying noncommensurate values of g_i and Z_i (i.e., values which do not satisfy (10)) in the calculation of Z_m by means of (7)). A Z_m can be calculated by using noncommensurate values of g_i and Z_i in (7), but a matching network designed from this Z_m will produce reflection coefficients g_1 and g_2 not in exact agreement with the values initially substituted into (7), when the network is terminated by Z_1 and Z_2.

IV. Phase Modulator

The symbols Q_g^2 and Q_Z^2 may be defined for the left and right sides of (10), respectively

$$Q_g^2=\frac{4|g_1-g_2|^2}{(|g_1|^2-1)(|g_2|^2-1)} \tag{11}$$

$$Q_Z^2=\frac{|Z_1-Z_2|^2}{R_1R_2}. \tag{12}$$

The rectangular coordinates of g_1 and g_2 may be written

$$g_i=|g_i|\cos\theta_i+j|g_i|\sin\theta_i \qquad (i=1,2). \tag{13}$$

Using these forms in (11) with Q_Z^2 for Q_g^2 leads to the result

$$|g_1|^2-2|g_1||g_2|\cos\phi+|g_2|^2=\frac{Q_Z^2}{4}(1-|g_1|^2)(1-|g_2|^2) \tag{14}$$

where $\phi=(\theta_1-\theta_2)$ is the phase difference between the two reflection coefficients. In the ideal phase shifter, a change in the phase of the output signal is required with signal amplitude held constant. For the reflection phase shifter this implies

$$|g_1|=|g_2|=|g|. \tag{14a}$$

Equation (14) provides a specific relationship between the relative signal amplitude available from a reflection-type phase shifter and the phase shift ϕ, for a given impedance pair. From (14) and (14a) this relation is

$$|g|=\left|\frac{m}{2}-\sqrt{1+\frac{m^2}{4}}\right| \tag{15}$$

where $m^2=8(1-\cos\phi)/Q_Z^2$. The minimum transfer loss in decibels of a reflection phase shifter is therefore $20\log|g|$. A typical dependence of relative signal amplitude on phase

[1]Assigning a zero value to the system determinant reduces the rank of its matrix to 3. This in principle allows 3 unknowns to be determined in terms of another; e.g., a/d, b/d, c/d. Then the condition for a reciprocal, lossless circuit: $ad+bc=1$, allows the determination of the fourth unknown.

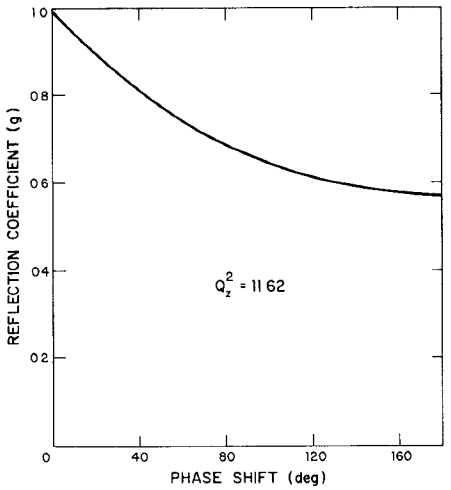

Fig. 2. Transfer factor g versus phase shift for a reflection phase shifter based on an impedance pair having $Q_Z^2 = 11.62$.

shift for the reflection-type phase shifter is shown in Fig. 2, which was calculated from (15) with $Q_Z^2 = 11.62$, corresponding to the 0.5 and 2.0-mA impedance pair in Table I, with (12).

V. Amplitude Modulator

In (14), the absolute phases of g_1 and g_2 do not appear, but only their difference ϕ. From (14), if the relative signal levels $|g_1|$ and $|g_2|$ at the output of an amplitude modulator are specified, the phase shift accompanying the modulation is then given by

$$\cos\phi = \frac{|g_1|^2 + |g_2|^2 - (Q_Z^2/4)(1-|g_1|^2)(1-|g_2|^2)}{2|g_1||g_2|}. \tag{16}$$

Solutions for a transformation network for the amplitude modulator do not exist for all arbitrarily chosen values of g_1 and g_2, but the existence of solutions and the associated phase shift ϕ may be found from (16) by trial. Alternatively, if a specific value of ϕ is assigned, the attainable $|g_2|$ for a given $|g_1|$ may be found from (14) to be

$$|g_2| = \left|G \pm \sqrt{F+G^2}\right| \tag{17}$$

where

$$G = \frac{|g_1|\cos\phi}{1+p_1^2}$$

$$F = \frac{p_1^2 - |g_1|^2}{1+p_1^2}$$

$$p_1^2 = \frac{Q_Z^2}{4}(1-|g_1|^2).$$

The assignment of $\phi = 0$ is a convenient choice for the amplitude modulator, implying an amplitude transition with no phase shift. A $\phi = 0$ always provides a solution for $|g_2|$ in (17).

VI. Examples of Modulator Design

As an example of the versatility of the procedure outlined in the preceding sections, it can be shown that a single pair of impedance states selected from Table I can be used with suitable transformation networks to produce a) an amplitude modulator, b) a 45° phase shifter, or c) a 90° phase shifter.

The impedance states from Table I which will be employed are

$$Z_1 = 35 - j11$$
$$Z_2 = 6 - j51.$$

A. Amplitude Modulator

The reflection coefficient magnitude (g_1) for the low-level state is arbitrarily assigned a value of 0.05. It may be assumed that no phase difference exists between g_1 and g_2. With no loss of generality, a phase angle of zero may be assigned to both. Therefore, using the Z_1 and Z_2 above, $g_1 = 0.05$ (scalar), and $Q_Z^2 = 11.62$, (17) may be employed to find: $g_2 = 0.875$. With these values, the solution of (A.14) to (A.19) is: $Z_m = (34.33 - j7.59)$. A solution for Z_m with negative real part also exists, but this is discarded since it is unrealizable by a passive network. A network is then required which will transform the Z_m of $(34.33 - j7.59)$ ohms to the system Z_0 of 50 Ω. This can be done by means of a section of transmission-line transformer of characteristic impedance 39.2 Ω and electrical length 122°. This impedance-transforming circuit is easily realizable in microstrip. The validity of the reflection-coefficient transformer can be verified by testing the Z_m value above in Kawakami's condition (1) rewritten for a Z_m-matching network

$$|g_t(Z_t')| = \left|\frac{Z_t - Z_m}{Z_t + Z_m^*}\right|. \tag{18}$$

Using the given Z_1 and Z_2 values, the transformed coefficients are confirmed to be $g_t = 0.05$, $g_2 = 0.875$, or an isolation of 26 dB and transmission loss of 1.2 dB. This confirmation, as cited in Section III, is the result of having selected g_t, g_2, Z_t, and Z_2 which satisfy (10).

B. 45° Phase Shifter

For a 45° phase shift with $Q_Z^2 = 11.62$, $|g_1| = |g_2| = 0.80$, from (15). Therefore, the minimum attenuation for the 45° phase shifter will be -1.94 dB. Suitable values of g_1 and g_2 for use in (7) are

$$g_1 = 0.80\angle 0° = 0.80 + j0.0$$
$$g_2 = 0.80\angle 45° = 0.566 + j0.566.$$

Using these data and the chosen Z_1, Z_2 in (7) produces the network parameter: $Z_m = (18.15 - j76.10)$. A simple matching network to transform this impedance to $Z_0 = 50$ Ω can be constructed of a single open-ended stub of electrical length 70° connected in shunt with the line at 75.8° from the termination.

C. 90° Phase Shifter

From (15), a 90° phase shifter with the Z_1 and Z_2 in use may have a reflection amplitude $|g_1|=|g_2|=g=0.668$, or -3.5-dB minimum loss. This data in (7) leads to a network parameter: $Z_m=24.48-j62.46$. This transformation can be realized with a single open stub of electrical length 62.6° connected at 77° from the termination.

V. Conclusion

A procedure has been given for deriving the design of a generalized two-state reflection-type modulator based on the transformed reflection coefficients of two-state variable impedances. The values of complex reflection coefficient available by transformation from a given two-state impedance pair are shown to be determined by a relation which occurs as a condition for the existence of a solution for the network parameters of the impedance transformers.

Appendix

The general network matrix for a lossless two-port circuit has the form

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} a & jb \\ jc & d \end{pmatrix} \tag{A.1}$$

where a, b, c, and d are real. The impedance transformation by the two-port is given by

$$Z_i' = \frac{aZ_i + jb}{jcZ_i + d}. \tag{A.2}$$

When (A.2) is introduced into the expression for reflection coefficient ((2) of the text) the result becomes

$$g_i(Z_i') = \frac{(a-jcZ_0)Z_i - (dZ_0 - jb)}{(a+jcZ_0)Z_i + (dZ_0 + jb)}. \tag{A.3}$$

I. Necessary Condition for Transformation Network

Equation (A.3) is equivalent to

$$(g_i-1)Z_i a + j(g_i-1)b + j(g_i+1)Z_0 Z_i c + (g_i+1)Z_0 d = 0. \tag{A.4}$$

Introducing the reflection variable σ

$$\sigma_i = \frac{g_i - 1}{g_i + 1}. \tag{A.5}$$

Equation (A.4) becomes

$$\sigma_i Z_i a + j\sigma_i b + jZ_0 Z_i c + Z_0 d = 0. \tag{A.6}$$

The complex form of σ_i and Z_i may be written

$$\sigma_i = \sigma_i' + j\sigma_i''$$
$$Z_i = R_i + jX_i$$

when these forms are introduced into (A.6) and the real and imaginary parts separated, the result is:

$$\text{Real: } (\sigma_i' R_i - \sigma_i'' X_i)a - \sigma_i'' b - Z_0 X_i c + Z_0 d = 0 \tag{A.7}$$

$$\text{Imag: } (\sigma_i'' R_i + \sigma_i' X_i)a + \sigma_i' b - Z_0 R_i c = 0. \tag{A.8}$$

Writing (A.7) and (A.8) for cases $i=1$ and $i=2$ yields a set of four homogeneous equations in the network constants a, b, c, d. These equations will have a unique solution only if the determinant of their coefficients vanishes. This determinant is

$$\begin{vmatrix} (\sigma_1' R_1 - \sigma_1'' X_1) & -\sigma_1'' & -Z_0 X_1 & Z_0 \\ (\sigma_2' R_2 - \sigma_2'' X_2) & -\sigma_2'' & -Z_0 X_2 & Z_0 \\ (\sigma_1'' R_1 + \sigma_1' X_1) & \sigma_1' & Z_0 R_1 & 0 \\ (\sigma_2'' R_2 + \sigma_2' X_2) & \sigma_2' & Z_0 R_2 & 0 \end{vmatrix} = 0. \tag{A.9}$$

Expanding (A.9) produces the result

$$\frac{(X_1 - X_2)^2 + (R_1 - R_2)^2}{R_1 R_2} = \frac{(\sigma_1' - \sigma_2')^2 + (\sigma_1'' - \sigma_2'')^2}{\sigma_1' \sigma_2'}.$$

This equation may be written more compactly in the form

$$\frac{|Z_1 - Z_2|^2}{R_1 R_2} = \frac{|\sigma_1 - \sigma_2|^2}{\sigma_1' \sigma_2'}. \tag{A.10}$$

Equation (A.10) is a necessary condition for the existence of a solution for the network constants of an impedance transformer which produces complex reflection coefficients g_1 and g_2 when terminated with impedances Z_1 and Z_2.

II. Solution for Network Parameter Z_m

Equation (A.3) may be written

$$g_i(Z_i') = \frac{Z_i - Z_m}{Z_i + Z_m^*} \frac{(a - jcZ_0)}{(a + jcZ_0)} = \frac{Z_i - Z_m}{Z_i + Z_m^*}\eta \tag{A.11}$$

where $Z_m = (dZ_0 - jb)/(a - jcZ_0)$ and η is a phase factor of unit amplitude: $\eta = \exp(2j\tan^{-1}(-cZ_0/a))$.

The versions of (A.11) for cases $i=1$ and $i=2$ may be combined

$$\frac{1}{\eta} = \frac{1}{g_1}\frac{Z_1 - Z_m}{Z_1 + Z_m^*} = \frac{1}{g_2}\frac{Z_2 - Z_m}{Z_2 + Z_m^*}. \tag{A.12}$$

The solution for Z_m^* is

$$Z_m^* = \frac{MZ_m + P}{Z_m + N} \tag{A.13}$$

where

$$M = -\frac{(g_1 Z_1 - g_2 Z_2)}{(g_1 - g_2)} \tag{A.14}$$

$$N = -\frac{(g_1 Z_2 - g_2 Z_1)}{(g_1 - g_2)} \tag{A.15}$$

$$P = Z_1 Z_2. \tag{A.16}$$

Equation (A.13) may be solved for Z_m by taking its complex conjugate and substituting back into itself. This leads to the solution

$$Z_m = -\alpha \pm \sqrt{\beta + \alpha^2} \tag{A.17}$$

where

$$\alpha = \frac{1}{2}\frac{P - P^* + NN^* - MM^*}{M + N^*}. \tag{A.18}$$

$$\beta = \frac{M^* P + NP^*}{M + N^*}. \tag{A.19}$$

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Analysis of Open-Type Dielectric Waveguides by the Finite-Element Iterative Method

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Abstract—**Dispersion characteristics for open-type dielectric waveguide structures operated at millimeter- and submillimeter-wave frequencies are calculated by a finite-element iterative procedure with a given criterion on the maximum field strength at the virtual boundary. Numerical results for a rectangular dielectric image guide are presented and compared with results from other methods. The strip dielectric guide and the insulated image guide with finite- or infinite-width substrates are also analyzed.**

I. INTRODUCTION

VARIOUS dielectric waveguide structures operated at millimeter-and submillimeter-wave frequencies have been recently developed [1]–[3]. In the structure design, it becomes important to calculate the dispersion characteristics, the field distributions, and other quantities. However, rigorous solutions have not been known except for specific structures [4]. Many approximate and numerical methods [1]–[10] for analyzing the various structures (see Fig. 1.) have been presented in the past decade. Among them, the effective dielectric constant method [1], [2], the transverse resonance method [5], [6], and other methods [3], [7] which cannot provide complete information on the field distributions, and it has been recently suggested [8] that [1], [2] are the single-mode approximations. The field-matching method [9] and the method employing the telegraphist's equation [10] are efficient where metallic walls are assumed. The influence of the metallic wall on the inherent

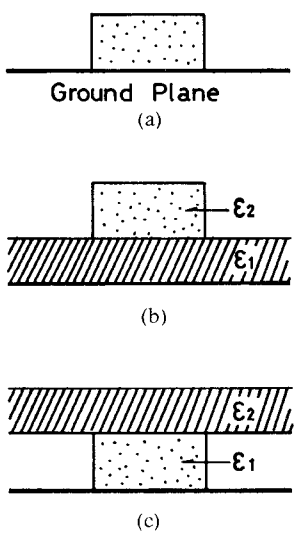

Fig. 1. Dielectric waveguide structures with conducting ground plane. (a) Dielectric image guide. (b) Strip dielectric guide for $\epsilon_1 > \epsilon_2$, insulated image guide for $\epsilon_1 < \epsilon_2$, single-material guide for $\epsilon_1 = \epsilon_2$. (c) Inverted strip dielectric guide for $\epsilon_1 \leqslant \epsilon_2$

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